Features of Some of Our Computer Algebra Applications

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Main Topics

• The beginning of our research in the field of Computer Algebra (CA).

• The features of some of our CA applications, considered in parallel with our collaboration with the Laboratory of Information Technologies (LIT) of JINR, strongly supported by V.P. Gerdt.
The Beginning

- Symbolic computations
  - Languages, algorithms, . . .
  - 1969 – 1970 Computing Center of Russian Academy of Sciences, Moscow, S. S. Lavrov
  
- Program systems for symbolic computations
- . . .

- REDUCE system, papers, . . .
  1079 – 1980 and later
  JINR, Dubna; R. N. Fedorova, later V.P. Gerdt

- Joint projects
- Conferences, workshops, . . .
Terminology and Some of Our Papers

- Аналитические преобразования
- Symbolic and algebraic computations
  . . .
- Computer Algebra
- Computer Algebra Systems (CAS)
- First our papers

- **Spiridonova, M.**: Languages for Symbolic Information Processing. J. Systems and Management (in Bulgarian), Sofia, 3-4, 29-34 (**1975**)
A Part of Our Presentations at Conferences and Workshops in Dubna

Геров А., Интегрированные численные и символьные вычисления на основе языка ALDES. Сб. трудов Международного совещания по аналитическим вычислениям на ЭВМ и их применению в теоретической физике, Дубна, 17-20 сентября, 1985, 28-32.

Спиридонова М., Организация и применение баз символьных преобразований на основе системы REDUCE 2, Сб. трудов Международного совещания по аналитическим вычислениям на ЭВМ и их применению в теоретической физике, Дубна, 17-20 сентября, 1985, 28-32.


...
Features of Our Research in the Field of Computer Algebra

**Computer Algebra:**

- **Algorithms, Systems, Applications**

- Use of the built-in capabilities of the general purpose Computer Algebra Systems (CAS) Reduce, SAC-2, Maple and Mathematica for solving problems.
- Development of special purpose CAS and program packages for solving specific problems. Joint work with the colleagues from the “application” areas.
- Conceptual and methodological research problems .
- Collaboration with Bulgarian and foreign research institutes (in Dubna, Dresden, Rome, Sophia Antipolis, ...).
- Other international activities.
- “Non-standard” applications.
- ...
Developed Special Purpose CAS and Program Packages

- Special Purpose CAS for linear algebra, based on SAC 2.
- Special Purpose CAS for Continued Fractions Manipulation, based on SAC 2.
- Reduce Package for Laplace Transformation.
- Reduce Package for Power Series Manipulation.
- Reduce Package for Investigation of Rational Functions.
- Reduce Package for Symbolic and Numerical Computations in Mathematical Geodesy.
- Two Legendre Transformation packages (with use of REDUCE and Maple).
- A collection of Mathematica packages for solving initial and boundary-value problems for some classes of differential equations, using the Operational Calculus Approach.
- . . .
Conceptual and Methodological Research Problems Related to the CA Applications

On the base of our experience and some features of the used CAS, we have developed the following conceptions:

(1) A conception of an Intelligent Computer Algebra System (ICAS).

It includes development of:

- language tools for more "natural" description of the problem to be solved;
- analysis of the problem description and choice of an appropriate algorithm from the "knowledge base" of the system;
- construction of a sequence of steps for full solution of the problem.

Importance of the mathematical knowledge representation . . .


. . .
(2) A Conception of an Applied Computer Algebra Microenvironment

Shortly about this conception:

• An Applied Microenvironment (AM) can be considered as a framework within a CAS which provides specific tools for solving a problem or a class of problems.

• The adjective "applied" points out that always such a microenvironment is developed for solving an applied problem. The particle "micro" indicates that it is a small framework in the large computational environment of the host general purpose CAS.

• It is supposed that being in an AM, one should be able to solve the formulated problem or a class of problems without the necessity other tools than those defined in it to be used. The last feature may not be valid for an usual library package. An AM should provide also facilities for interaction with the user, for delivering information about the used method, for explanation of the results, etc.

• Some experimental implementations of this conception are developed.

Our International Activities

- Joint projects with research institutions in Russia, Hungary, Italy, France, . . .
  - Regular joint projects with LIT of JINR, Dubna, supported by V.P. Gerdt.
  - Participation of V.P. Gerdt in Conferences and seminars of Bulgaria.

- Participation in International Research groups.
- Participation in International Workshops and Conferences.
- Organization of International Workshops and Conferences.

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Working Group 18 (WG 18) named “Knowledge Representation in Man-machine and Robotic Systems”

• Papers stimulated by our participation in these groups:

Геров А., Ю. Капитонова, М. Спиридонова, В. Томов, Интеллектуальные системы аналитических преобразований, Представление знаний в человеко-машинных и робототехнических системах, Том С: Прикладные человеко-машинные системы, ориентированные на знания, ВЦ АН СССР, ВИНИТИ, Москва, 1984, 112-136. (WG 18)

Томов В., А. Геров, Интеллектуальные прикладные программные системы, Сб.трудов Междунар. семинара „Проблемы и применения искусственного интеллекта”, Варна, 21-25 сентября, 1987, 58-66

Спиридонова М., М. Нишева, Интеллектуализированные системы аналитических преобразований и их применение в научных исследованиях и проектировании, Сб. трудов Междунар. семинара „Теория и применение искусственного интеллекта”, Созопол, 29 мая – 2 июня, 1989, 313-318.

...
Our membership of ACA WG (Applications of Computer Algebra Working Group) - from 2005 (encouraged by V.P. Gerdt).

Organization of two ACA Conferences in Bulgaria:

• ACA 2006 - in Varna
  (with Gen. Chairs V. P. Gerdt, M. Spiridonova)
• ACA 2012 – in Sofia
  (with Gen. Chair M. Spiridonova)
More of Our Papers

*(giving information about some of our research topics)*


...
More Papers


...
More Papers


Nonstandard Applications

Experimental application of the CAS Mathematica for solving a system of nonlinear algebraic equations, related to a telecommunication problem.

The first attempt was presented in Dubna in 2014:

SOME RESULTS RELATED TO THE ANALYTICAL MODELLING OF TELECOMMUNICATION SYSTEMS

- Analytical model of overall telecommunication system with queuing.
- Service qualifications.
- Causal service structure of a virtual device.
- Three intuitionistic fuzzy service characterizations.
- Intuitionistic fuzzy characterizations of consecutive composition of services.
BASE VIRTUAL DEVICE TYPES

- **Director** – unconditionally points to the next device, which the request shall enter;
- **Terminator** – eliminates from the model every request entered;
- **Server** – models traffic and time characteristics of the requests processing;
- **Switch** (Transition) selects one of its possible exits for each request entered;
- **Causal device** – virtual device defined for presentation of causes of service ending, e.g. successful (carried) or not (interrupted, abandoned, etc.);
- **Fictive device** – device presenting fictive traffic which is necessary for engineering. For example, not carried traffic is fictive, but is used for calculating the equivalent offered traffic, which is necessary for device dimensioning.

Fig. 1. Graphical representation of different functional types of base virtual devices.
PARAMETERS OF THE BASE VIRTUAL DEVICES

Every base virtual device has 6 parameters which characterize the flow.

- $F_x$ – intensity or incoming rate (frequency) of the flow of requests to device $x$;
- $P_x$ – probability of directing the requests towards device $x$;
- $T_x$ – service time (duration of servicing of a request) in device $x$;
- $Y_x$ – traffic intensity [Erlang];
- $V_x$ – traffic volume [Erlang - time unit];
- $N_x$ – number of lines (service resources, positions, capacity) of device $x$. 
CONCEPTUAL MODEL OF OVERALL TELECOMMUNICATION SYSTEM WITH QUEUING 1/2

(Andonov et al., 2019)

Fig. 2. Stages Dialing and Switching
CONCEPTUAL MODEL OF OVERALL TELECOMMUNICATION SYSTEM WITH QUEUING 2/2

(Andonov et al., 2019)

Fig. 3. Stages Ringing and Communication
Analytical model of overall telecommunication system with queuing (1/6)
(Andonov et al, 2019)

Unknown parameters (15):

• $F_0; Yab; Fa; Pbws; Tws; Pbr; dem.Fa; rep.Fa; ofr.Fws; crr.Fs; Ts; Fs; Tcs; Ys; p_0;$

Generalized static parameters (12):

• $S_1; S_2; S_3; S_4; S_5; S_6; R_1; R_2; R_3; S_{1z}; S_{2z}; S_{3z};$

\[ Y_{ab} = Fa[S_1 + S_2 Pbws + S_3(1 - Pbws)Tws + S_4(1 - Pbws)Tcs + S_5 Pbr + S_6 Pbr Pbws]. \]  \tag{61}

\[ Fa = dem.Fa + rep.Fa. \]  \tag{62}

\[ dem.Fa = F_0(N_{ab} + MY_{ab}). \]  \tag{63}

\[ rep.Fa = Fa[R_1 + R_2(1 - Pbws)Pbr + R_3 Pbws], \]  \tag{64}
Analytical model of overall telecommunication system with queuing (2/6)

\[
P_{br} = \begin{cases} 
\frac{Y_{ab} - 1}{N_{ab} - 1}, & \text{if } 1 < Y_{ab} \leq N_{ab}. \\
0, & \text{if } 0 \leq Y_{ab} \leq 1. 
\end{cases} \quad (65)
\]

\[
ofr.F_{ws} = Fa(1 - Pad)(1 - Pid). \quad (66)
\]

\[
crr.F_{s} = ofr.F_{ws}(1 - Pbws)(1 - Pis)(1 - Pns). \quad (67)
\]

\[
F_{s} = Fa(1 - Pad)(1 - Pid)(1 - Pbws). \quad (68)
\]

\[
Y_{s} = FsT_{s}. \quad (69)
\]

\[
P_{bws} = \frac{\lambda^{N_{s} + N_{ws}}}{N_{s}^{N_{ws}} N_{s}! \mu^{N_{s} + N_{ws}} p_{0}}. \quad (70)
\]
Analytical model of overall telecommunication system with queuing (3/6)

\[
p_0^{-1} = \begin{cases} 
\sum_{n=0}^{N_s-1} \frac{r^n}{n!} + \frac{r^N_s}{N_s!} \cdot \frac{1-\rho^{N_{ws}+1}}{1-\rho}, & \text{if } \rho \neq 1. \\
\sum_{n=0}^{N_s-1} \frac{r^n}{n!} + \frac{r^N_s}{N_s!} (N_{ws} + 1), & \text{if } \rho = 1.
\end{cases}
\]  

(71)

\[
T_{ws} = \frac{p_0^2 (N_s \rho r)^N_s \rho (1 - \rho^{N_{ws}}) \cdot ((\rho - 1)\rho^{N_{ws}} (N_{ws} + 1) + 1 - \rho^{N_{ws}+1})}{(N_s!)^2 (1 - \rho)^3 \lambda (1 - P_{bw})}.
\]  

(72)

\[
T_s = S_{1z} + S_{2z} P_{br} + S_{3z} T_{cs}.
\]  

(73)

\[
T_{cs} = T_{cs}^* + P_{br} T_{br} + (1 - P_{br}) T_b.
\]  

(74)
Analytical model of overall telecommunication system with queuing (4/6)

\[ \lambda = ofr.Fws, \quad (75) \]

\[ \mu = \frac{1}{T_s}, \quad (76) \]

\[ r = \frac{\lambda}{\mu} = ofr.FwsT_s, \quad (77) \]

\[ \rho = \frac{r}{N_s} = \frac{ofr.FwsT_s}{N_s}, \quad (78) \]

\[ T_b = ParTar + (1 - Par)[Tcr + PacTac + (1 - Pac)Tcc], \quad (79) \]

\[ S_1 = Ted + PadTad + (1 - Pad)[PidTid + (1 - Pid)[Tcd + PisTis + (1 - Pis)
\quad .[PnsTns + (1 - Pns)2Tb]]], \quad (80) \]
Analytical model of overall telecommunication system with queuing (5/6)

\[
S_2 = (1 - P_{ad})(1 - P_{id})[T_{bw} - P_{is}T_{is} - (1 - P_{is})[P_{ns}T_{ns} - (1 - P_{ns})2T_{b}]], \quad (81)
\]

\[
S_3 = (1 - P_{ad})(1 - P_{id}), \quad (82)
\]

\[
S_4 = (1 - P_{ad})(1 - P_{id})(1 - P_{is})(1 - P_{ns}), \quad (83)
\]

\[
S_5 = (1 - P_{ad})(1 - P_{id})(1 - P_{is})(1 - P_{ns})(T_{br} - 2T_{b}), \quad (84)
\]

\[
S_6 = -T_{br}(1 - P_{ad})(1 - P_{id})(1 - P_{is})(1 - P_{ns}), \quad (85)
\]

\[
R_1 = P_{ad}P_{rad} + (1 - P_{ad})[P_{id}P_{rid} + (1 - P_{id})[P_{is}P_{ris} + (1 - P_{is})[P_{ns}P_{ns}\]
\quad + (1 - P_{ns})[P_{ar}P_{ar} + (1 - P_{ar})[P_{ac}P_{ac} + (1 - P_{ac})P_{cc}]])], \quad (86)
\]

\[
R_2 = (1 - P_{ad})(1 - P_{id})(1 - P_{is})(1 - P_{ns})[P_{br} - P_{ar}P_{ar} - (1 - P_{ar})[P_{ac}P_{ac}\]
\quad + (1 - P_{ac})P_{cc}], \quad (87)
\]
Analytical model of overall telecommunication system with queuing (6/6)

\[
R_3 = (1-Pad)(1-Pid)[Prbws-[PisPris+(1-Pis)[PnsPrns+(1-Pns)[ParPrar+
+(1-Par)[PacPrac+(1-Pac)Prcc]]]],
\]

(88)

\[
S_{1z} = PisTis + (1-Pis)[PnsTns + (1-Pns)(Tb)],
\]

(89)

\[
S_{2z} = (1-Pis)(1-Pns)(Tbr-Tb),
\]

(90)

\[
S_{3z} = (1-Pis)(1-Pns).
\]

(91)
Definition 1: The served traffic in a pool of resources is the traffic occupying (using) resources in the pool.

Definition 2: The carried traffic in a pool of resources is the traffic which was successfully served in the pool (and carried to the next service phase).

Definition 3: The parasitic traffic in a pool of resources is the traffic which was unsuccessfully served in the pool.

In the above proposed definitions “served” and “carried” traffic are different terms. While in the ITU-T Rec. E.600 def. (Term 5.5) the carried traffic is “the traffic served by a pool of resources”. We believe that this differentiation allows a better and more detailed traffic and QoS characterization.
CAUSAL CLASSIFICATION

Special qualifiers are used to characterize the flow intensity (F) and the traffic (Y):

• prs. – comes from “parasitic”. Used to denote the parasitic flow and traffic;
• srv. – comes from “served”. Used to denote the served flow of requests;
• nsr. – comes from “not served”. It denotes a fictive traffic;
• blc. – comes from “blocked”. Denotes the blocked traffic;
• rep. – comes from “repeated”. Denotes repeated flow of requests;
• crr. – comes from “carried”;
• eff. - comes from “effective”.

PROPOSED INTUITIONISTIC FUZZY CHARACTERIZATION

• Successful – a target service which belongs to a fully completed successful service. It corresponds to carried. The qualifier corresponding to it is scc.

• Uncertain – uncompleted target service or target service without clear result, due to any reason. The qualifier corresponding to it is unc.

• Unsuccessful (not served) – a service which does not belong to the desired successful service. It is not completed due to some reason, for instance overloaded equipment; denial of the patient; invalid documents; contraindications etc. The qualifier corresponding to it is nsr.

The above characterizations allow for the quantifying of the uncertainty in the services through the use of Intuitionistic Fuzzy Pairs (IFPs), (Poryazov et al. 2020a).
Fig. 4. Causal decomposition of the traffic inside a service device $x$. 
FIRST IF SERVICE CHARACTERIZATION

Using the proposed causal decomposition of the traffic inside a service device $x$, we define three IF characterizations (Poryazov et al., 2020b). The first is the *IF traffic characterization*. We define an IFP $<\mu_x^y, \nu_x^y>$ (Atanassov et al., 2013), where

\[
\mu_x^y = \frac{sec.Yx}{ofr.Yx}, \quad \nu_x^y = \frac{abr.Yx}{ofr.Yx}.
\]

The degree of uncertainty $\pi_x^y$ is given by:

\[
\pi_x^y = \frac{unc.Yx}{ofr.Yx}.
\]

From the graphical representation (Fig. 4) it is obvious that the so defined pair $<\mu_x^y, \nu_x^y>$ is indeed an IFP.
The second IF characterization is the *IF flow characterization*. We define an IFP $\langle \mu_x^f, \nu_x^f \rangle$, where

\[
\mu_x^f = \frac{scc邢x}{ofr邢x} \quad ; \quad \nu_x^f = \frac{abr邢x}{ofr邢x}.
\]

The degree of uncertainty $\pi_x^f$ is given by:

\[
\pi_x^f = \frac{unc邢x}{ofr邢x}.
\]

From the graphical representation (Fig. 4) it is obvious that the so defined pair $\langle \mu_x^f, \nu_x^f \rangle$ is indeed an IFP.
The third proposed characterization is the *IF time characterization*. We define the pair $<\mu_x^t, \nu_x^t>$, where

$$
\mu_x^t = \frac{\text{prt.scc.Tx}}{\text{srv.Tx}},
\nu_x^t = \frac{\text{prt.abr.Tx}}{\text{srv.Tx}}.
$$

The degree of uncertainty $\pi_x^t$ given by:

$$
\pi_x^t = \frac{\text{prt.unc.Tx}}{\text{srv.Tx}}.
$$
CONSECUTIVE COMPOSITION OF SERVICES

Fig. 5. Conceptual model of a consecutive composition of services in a comprise service device x. (Poryazov et al., 2020b)
Using the graphical representation (Fig. 5), the methods of teletraffic theory and the probability theory, we derive the following equations:

\[
\text{sc. } Yx = \text{sc. } Fx (\text{sc. } T_1 + \text{sc. } T_2).
\]

\[
\text{sc. } Fx = \text{ofr. } Fx (1 - \text{nsr. } P_1)(1 - \text{unc. } P_1)(1 - \text{nsr. } P_2)(1 - \text{unc. } P_2).
\]

\[
\text{ofr. } Yx = \text{ofr. } Fx (\text{unc. } P_1 \text{ unc. } T_1 + (1 - \text{unc. } P_1)[\text{sc. } T_1 + \text{unc. } P_2 \text{ unc. } T_2 \\
+ (1 - \text{unc. } P_2) \text{sc. } T_2]).
\]
Using the above expressions equations, for the membership degree (the degree of validity) of the IFP for the comprise service device $x$, we obtain:

$$\mu_x^y = \frac{(1 - \text{nsr}.P_1)(1 - \text{unc}.P_1)(1 - \text{nsr}.P_2)(1 - \text{unc}.P_2)}{\text{unc}.P_1 \text{unc}.T_1 + (1 - \text{unc}.P_1)[\text{sec}.T_1 + \text{unc}.P_2 \text{unc}.T_2 + (1 - \text{unc}.P_2)\text{sec}.T_2]} \cdot (18)$$
In order to obtain an expression for $\nu_x^y$, first we derive the following equations:

\[
abr.Yx = nsr.Y_1 + unc.Y_1 + nsr.Y_2 = 
\]

\[
nsr.F_1 srv.T_1 + unc.F_1 unc.T_1 + nsr.F_2 (sec.T_1 + srv.T_2). \quad (19)
\]

\[
\nu_x^y = \frac{nsr.F_1 srv.T_1 + unc.F_1 unc.T_1 + nsr.F_2 (sec.T_1 + srv.T_2)}{A}, \quad (20)
\]

where

\[
A = ofr.Fx[unc.P_1 unc.T_1 + (1 - unc.P_1)[sec.T_1 + unc.P_2 unc.T_2 \\
+(1 - unc.P_2)sec.T_2]] \quad (21)
\]
To determine the degree of uncertainty $\pi_{x}^{y}$, we take into account the fact that the uncertain traffic and flow intensities of the comprise device are equal to those of the second embedded device, i.e.,

\[
\text{unc.} \ Y_{x} = \text{unc.} \ Y_{2} = \text{unc.} \ F_{x} \ \text{unc.} \ T_{2}.
\]  

(22)

\[
\text{unc.} \ F_{x} = \text{unc.} \ F_{2} = \text{oofr.} \ F_{x} (1 - \text{nsr.} \ P_{1})(1 - \text{unc.} \ P_{1})(1 - \text{nsr.} \ P_{2})\text{unc.} \ P_{2}.
\]  

(23)

\[
\pi_{x}^{y} = \frac{(1 - \text{nsr.} \ P_{1})(1 - \text{unc.} \ P_{1})(1 - \text{nsr.} \ P_{2})\text{unc.} \ P_{2} \text{unc.} \ T_{2}}{\text{unc.} \ P_{1} \text{unc.} \ T_{1} + (1 - \text{unc.} \ P_{1})[\text{sec.} \ T_{1} + \text{unc.} \ P_{2} \text{unc.} \ T_{2} + (1 - \text{unc.} \ P_{2})\text{sec.} \ T_{2}]}.
\]  

(24)
IF FLOW CHARACTERIZATION IN THE CASE OF CONSECUTIVE COMPOSITION

\[
\mu^f_x = (1 - \text{nsr.}P_1)(1 - \text{unc.}P_1)(1 - \text{nsr.}P_2)(1 - \text{unc.}P_2). \quad (26)
\]

\[

\nu^f_x = \text{nsr.}P_1 + (1 - \text{nsr.}P_1)[\text{unc.}P_1 + (1 - \text{unc.}P_1)\text{nsr.}P_2]. \quad (27)
\]

\[
\pi^f_x = (1 - \text{nsr.}P_1)(1 - \text{unc.}P_1)(1 - \text{nsr.}P_2)\text{unc.}P_2. \quad (28)
\]
IF TIME CHARACTERIZATION IN THE CASE OF CONSECUTIVE COMPOSITION

\[
\mu^t_x = \frac{(1 - unc.P_1)(1 - unc.P_2)(sec.T_1 + sec.T_2)}{unc.P_1 + (1 - unc.P_1)} \cdot \frac{1}{B},
\]

where

\[
B = unc.P_1 unc.T_1 + (1 - unc.P_1)[sec.T_1 + (1 - nsr.P_2)[unc.P_2 unc.T_2 + (1 - unc.P_2)sec.T_2]].
\]

\[
\nu^t_x = \frac{unc.P_1 unc.T_1 + (1 - unc.P_1)nsr.P_2 sec.T_1}{C},
\]

where

\[
C = (1 - nsr.P_1)[unc.P_1 unc.T_1 + (1 - unc.P_1)[sec.T_1 + (1 - nsr.P_2)[unc.P_2 unc.T_2 + (1 - unc.P_2)sec.T_2]]].
\]

\[
\pi^t_x = \frac{(1 - unc.P_1)(1 - nsr.P_2)unc.P_2 unc.T_2}{D},
\]

where

\[
D = (1 - nsr.P_1)[unc.P_1 unc.T_1 + (1 - unc.P_1)[sec.T_1 + (1 - nsr.P_2)[unc.P_2 unc.T_2 + (1 - unc.P_2)sec.T_2]]].
\]
CONCLUSIONS

- Three intuitionistic fuzzy characterizations of the uncertainty in a service device are defined: IF traffic characterization, IF flow characterization and IF time characterization. They are defined in the form of IFPs.

- Analytical expressions for the three proposed characterizations are obtained in the case of a composition of two consecutively connected devices.

- It is not easy to express the degrees of validity, non-validity and uncertainty in comprise service devices, consisting of a large number of base virtual devices. In such cases, estimations for the comprise devices can be obtained through aggregation or the operations defined over IFPs.

- In order to evaluate the proposed IF characterizations for highly complex compositions of services such as in the overall telecommunication system, the corresponding analytical model has to be derived and solved using appropriate computer algebra algorithms and numerical methods.
REFERENCES


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